1. Let $I = [0,1]$. Let $g : I \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{1}{q} & \text{if } x \in \mathbb{Q} \text{ and } x = \frac{p}{q},\ \text{g.c.d.}(p,q) = 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that g is Riemann Integrable.

2. Let $v \in \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{x_1 x_2^4}{x_1^4 + x_2^6} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Find the directional derivative in the direction v of g at 0.
(b) Is g differentiable at 0 ?

3. Decide whether each of the following statements are true or false providing adequate justification.

(a) Let $a < b$ be two real numbers and $f : [a,b] \to \mathbb{R}$ is a bounded function. If f^2 is Riemann integrable then f is Reimann integrable.

(b) Let (X,d) be a metric space and $E \subset X$. If $\bar{E}$ is connected then E is connected.

4. Let $\mathbb{R}^2$ be equipped with the following metric d where $d(x,y)$ is the number of coordinates where x and y differ. For example:

$$d\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 300 \end{bmatrix}\right) = 1$$

(a) Show that $(\mathbb{R}^2, d)$ is a metric space.
(b) Describe the open balls of radius ϵ around the origin in $(\mathbb{R}^2, d)$ for all $\epsilon > 0$
(c) Show that every subset of $\mathbb{R}^2$ is open w.r.t d and describe which functions $f : (\mathbb{R}^2, d) \to (\mathbb{R}^2, d)$ are continuous.

5. The base of a rectangular box costs three times as much per square foot as do the sides and top. Find the relative domensions for the most economical box of given volume.